



Design and Off-Design Calculations of Single Spool Turbojet Engine Performance

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Abstract: The research project of the Open University and the National Research and Innovation Agency is taking place at the Open University, to design a turbojet engine that produces the targeted thrust at a speed (M), the height of the pilot with the rotational speed (RPM) set in the research activity. This paper presents the design of the performance of a single spool turbojet engine at Design Point (DP) and Off-Design Point (ODP) conditions. The calculation of the DP condition is solved using equations that have been provided in the literature, to obtain the appropriate mass flow rate. In the calculation of ODP with variations in temperature, pressure, Mach numbers using the C.N.Reffold semi-dimensional mass flow (SDMF) method to find the compressor pressure ratio. The purpose of this article is to present an ODP calculation that avoids the use of "nested" repetitive calculations and simplifies the solution process by using manual calculations with a success rate of less than 5%.

Keywords: Single spool, design, off-design. performance, turbojet

NOMENCLATUR

A	area	T	temperature
C	velocity	TET	turbine entry temperature
CR	compressor pressure ratio	TTR	turbine temperature ratio
C_p	specific heat at constant pressure	\dot{w}	power (work rate)
C_v	specific heat at constant volume		
D	diameter		
Δ	delta, increase or decrease		
DP	design point	<i>Subscripts</i>	
F	thrust	1-5	Station number (see Fig.1)
γ	gamma, ratio of specific heats, C_p/C_v		
h	specific enthalpy		
η	eta, efficiency		
k	a constant		
LCV	low calorific value (specific energy)		
M	flight Mach Number		
\dot{m}	mass flow rate (air, gas, fuel)		
OPL	overall pressure loss (combustor)		
P	pressure		
ρ	rho, density		
R	specific gas constant		
RPM	rotational speed in rev/min		
$SDMF$	semi-dimensional mass flow		
SFC	specific fuel consumption		

Received: October 30, 2024; Revised: November 30, 2024; Accepted: December 18, 2024;

Published: December 30, 2024

1. INTRODUCTION

The purpose of the Design Point (DP) calculation is to obtain estimates of the performance parameters (primarily specific thrust and specific fuel consumption) in terms of design constraints (such as maximum allowable turbine temperature and achievable component efficiency), flight conditions (ambient pressure, temperature, (Mustafa Cavcar) and Mach number), and design options such as compressor pressure ratio,

While the Off-Design Point (ODP) calculation is to obtain the mass air flow rate based on flight conditions (altitude and mach number) design conditions and turbojet engine performance and the main Off-Design Point (ODP) design performance is to obtain the turbojet engine thrust.

According to Jack D. Mattingly, etc., 2002; H.I.H. Saravanamuttoo, etc., 2017. dimensions and performance are expressed in ratios. The pressure ratio that for any given maximum cycle temperature will provide the overall efficiency, and mass flow required to deliver the desired power. Designing each component of a turbojet engine so that the complete unit will deliver the required performance when operating at the design point; that is, when operating at the specific speed, pressure ratio, and mass flow for which the component is designed.

Turbojet engine cycle analysis calculations are carried out in 2 (two) ways, namely performance modeling at the design point and performance modeling outside the design point. When performing design point performance modeling, important parameters required are to determine the altitude and Mach number of engine operation and to estimate the compressor ratio and air mass flow rate to be able to produce the targeted thrust. The results of the design point calculation are used as a reference for calculating off-design point performance modeling (ODP).

In this paper, the DP calculation uses equations that already exist in the literature, while to complete the ODP calculation, the method from C.N.Reffold is used, namely the semi-dimensional mass flowrate method to determine the compressor pressure ratio to avoid the use of 'nested' iterative calculations that simplify the solution process.

2. LITERATURE REVIEW

A turbojet engine is a gas turbine engine that works by compressing air through the inlet and compressor (centrifugal), mixing fuel with compressed air, burning the mixture in the combustion chamber, then flowing hot, high-pressure air through a turbine and nozzle as shown in Figure 1.

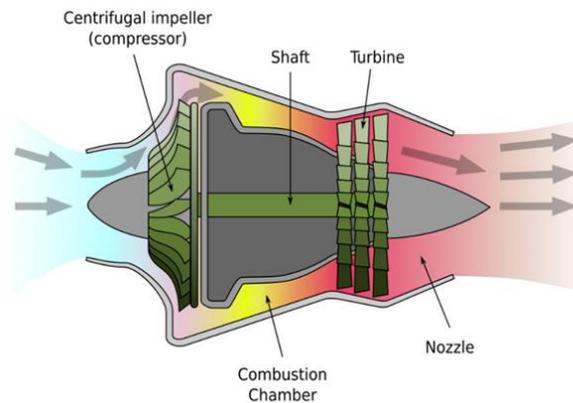


Figure 1. Schematic diagram showing the operation of a centrifugal flow turbojet engine.
Source: https://commons.wikimedia.org/wiki/File:Turbojet_operation_centrifugal_flow-en.svg

The working principle of a turbojet engine is that air is taken through the front inlet of the engine (state level 1). Then it is compressed until there is an increase in pressure where for a single stage it is between 3 to 4 times the inlet pressure of state level 2. The process of changing the thermodynamic state level 1-2 is ideally isentropic. The compressor output that produces pressurized air enters the combustion chamber and is then mixed with fuel at a certain ratio to be burned so that there is an increase in temperature so that it expands before being passed to the turbine section (state level 3). The process of changing the thermodynamic state level 2-3 is ideally isobar. The turbine will rotate after receiving the designed combustion chamber flow

to meet the power requirements and rotational speed required by the compressor (state level 4). The combustion gas is expanded isentropically in the turbine (process 3-4). The exhaust gas after passing through the turbine section will pass through a nozzle to produce thrust due to the reactive power of the exhaust gas flow. The schematic flow diagram and main components are shown in figure 2(a). The ideal thermodynamic cycle

in the Pressure-Volume (p - v) diagram and the Temperature-Entropy (T - s) diagram are shown in figures 2(b) and 2(c)

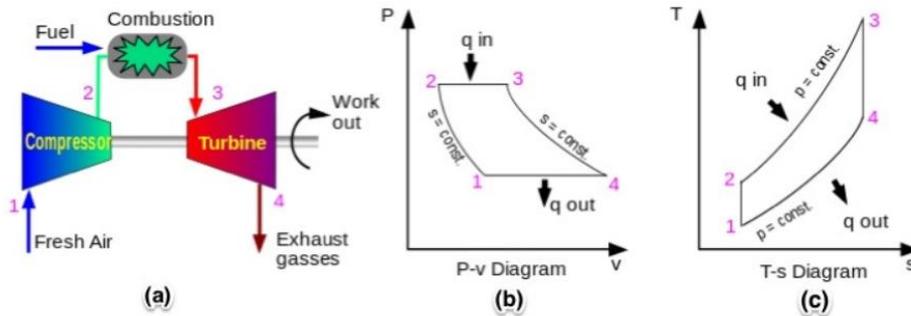


Figure 2. (a) Brayton Cycle Schematic (b) Brayton Cycle P-V Diagram (c) Brayton Cycle T-s Diagram

Source : <https://artikel-teknologi.com/siklus-brayton/>

DP and ODP calculations of single spool turbojet performance models. Many studies have been conducted for turbojet model calculations such as Dominik Klein, Chamil Abeykoon, (2015) developed a gas turbine model to improve efficiency in the turbine channel by increasing the temperature to 2000 °C by developing new materials and designing components as aerodynamic as possible using commercial software.

Andrea LAZZARETTO* and Andrea TOFFOLO, (2001) analysis of the DP turbojet model was calculated using the thermodynamic cycle method while for the off-Design Point analysis it refers to the compressor map and turbine map. The mapping results were used to calculate using commercial software.

3. METHODS

Turbojet is a jet engine that produces thrust by ejecting high-energy gas from the exhaust nozzle.

A turbojet engine is a gas turbine engine designed to convert thermal energy into mechanical energy on the turbine shaft to rotate the compressor and produce thrust. The thrust is generated from the fuel combustion process.

A turbojet has 5 (five) main components, namely intake (1), compressor (2), combustion chamber (3), turbine (4) and nozzle (5) which are shown in Figure 3. This figure shows the engine schematic with the numbering convention used in the calculation.

The numbers written in smaller letters at each condition indicate pressure and temperature. For example, T4 is the stagnation or total temperature at the turbine entry. Calculations on the condition of the main components with stagnation or total conditions while the inlet and outlet are in static conditions

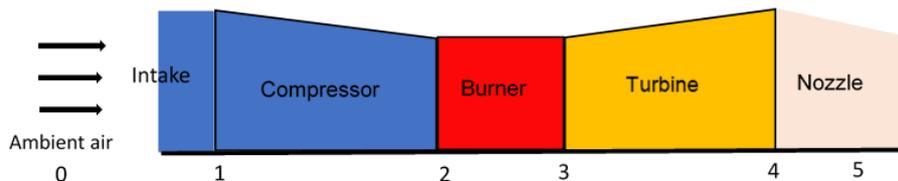


Figure 3. Schematic diagram of a turbojet engine

Design Point (DP)

First, determine the design point flight altitude and mach number. To determine the air temperature (P0) and air pressure (T0) conditions at the specified flight altitude referring to the International Standard Atmospheric (Mustafa Cavcar).

Second, determine the important targets and pressure loss parameters as follows:

- Mechanical efficiency for the main shaft and combustion efficiency for the combustor
- Polytopic efficiency for the compressor and turbine
 - Pressure ratio for the inlet
 - Maximum allowable overall pressure loss (OPL) for the combustor.

Usually expressed as a percentage of the inlet pressure

Third, find the air and fuel data. Cp for air and γ (specific heat ratio), vary as a function of temperature. Computational methods for calculation are available (Gordon-McBride) but are not required for simple models. The following values;

- Specific gas constant R is 287 J/kgK
- Cp values are 1005 J/kgK for the cold section and 1148 j/kgK for the hot section.
- γ can be found for any value of Cp because $\gamma = C_p / C_v = C_p / (C_p - R)$
- Fuel consumption values are usually taken as the low calorific value (LCV) in gas turbine modeling. A typical value for kerosene type fuel is 43 MJ/kg

The four designers must make key engine design choices before modeling engine performance: by setting the turbine inlet temperature (TET) limited by material capabilities and assuming an estimated compressor pressure ratio (CR).

Dynamic restoration

The initial calculation begins with the restoration of the air pressure and temperature to the total conditions. As the air flow stagnates from the true airspeed (TAS) to zero relative speed, the temperature and pressure increase as a function of the airspeed (and Mach Number, M).

TAS (C_a) needs to be evaluated for the thrust calculation and can be found by:

$$C_a = M\sqrt{\gamma RT_a} \quad (1)$$

Jack D. Mattingly; William H. Heiser; David T. Pratt, Turbojet starting in free stream with total isentropic condition calculation

$$\frac{T_0}{T_a} = \left[1 + \left(\frac{\gamma-1}{2} \right) M^2 \right] \quad (2)$$

$$\frac{P_0}{P_a} = \left[\frac{T_0}{T_a} \right]^{\gamma/(\gamma-1)} \quad (3)$$

Intake

The performance of the intake system can be modeled using the US Department of Defense military specification MIL-E-5008 (C.N.Reffold):

If $M > 1$ then

$$\frac{P_1}{P_0} = \left(\frac{P_1}{P_0} \right)_{max} \{ 1 - 0.075 (M - 1)^{1.35} \} \quad (4)$$

If Not

$$\frac{P_1}{P_0} = \left(\frac{P_1}{P_0} \right)_{max} \quad (5)$$

Assuming the input process (diffuser) is isentropic, then $T_1 = T_0$.

Compressor

CR(P_2/P_1) has been decided for the compressor. Now, to determine the temperature rise using the polytropic efficiency:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma\eta_{poly}} \quad (6)$$

The power consumed by the compressor is determined by calculating the enthalpy increase:

$$W_c = \dot{m} C_p \Delta T \quad (7)$$

At the initial stage we can calculate the specific power of the compressor by eliminating the mass flow term.

The adiabatic efficiency of the compressor is determined by the following equation:

$$\eta_{cs} = \frac{(C_R^{(\gamma-1)/\gamma-1})}{\left(\frac{T_2}{T_1}-1\right)} \quad (8)$$

Jet pipe and nozzle

If friction losses are neglected in the jet pipe, then the total temperature and pressure at station 5 will be the same as the temperature and pressure at station 4.

To calculate the static pressure at the exit, P_{s5} , we first calculate the expansion pressure P_{cric} at the choke condition ($M = 1$). If this pressure is less than the static air pressure, then the nozzle will expand the flow only up to the static air pressure ($M < 1$).

The critical pressure is:

$$P_c = P_5 \left[1 - \left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\gamma/(\gamma-1)} \quad (17)$$

and if this pressure is greater than the static air pressure, P_a , then the outlet flow is choked and will be the exit static pressure, P_{s5} .

If the critical pressure is less than P_a , then the nozzle will only allow expansion up to P_a and therefore, there will be no pressure thrust component.

The total temperature at the nozzle exit, T_5 , is the same as the temperature at the turbine exit. However, if the flow is choked, the static temperature can be found as follows:

$$T_{s5} = T_5 \left(\frac{2}{\gamma+1}\right) \quad (18)$$

If the flow is unimpeded, the outlet static temperature is determined by:

$$T_{s5} = T_5 \left(\frac{P_a}{P_5}\right)^{(\gamma-1)/\gamma} \quad (19)$$

The static equation allows us to find the density at the exit:

$$\rho_{s5} = \frac{P_{s5}}{(RT_{s5})} \quad (20)$$

We can find the exit velocity for choked flow by:

$$C_5 = \sqrt{\gamma RT_{s5}} \quad (21)$$

If the flow is not blocked, then:

$$C_5 = \sqrt{2C_p(T_5 - T_{s5})} \quad (22)$$

PERFORMANCE CALCULATION

The pressure component of the thrust is $A(P_{s5} - P_a)$ where A is the nozzle exit area. To find the specific thrust due to pressure we must divide by the mass flow rate. So, since $A = \dot{m}/\rho C_5$, the pressure component of the specific thrust is given by $(P_{s5} - P_a)/(\rho_{s5} C_5)$.

The momentum component of the thrust is $\dot{m}(C_5 - C_a)$ and we divide by \dot{m} to get the specific thrust due to momentum. So, the total F_s is given by:

$$F_s = \frac{(P_{s5} - P_a)}{(\rho_{s5} C_5)} + (C_5 - C_a) \quad (23)$$

And

$$SFC = \frac{\dot{m}_f}{F} = \frac{FAR \dot{m}_{air}}{F_s \dot{m}_{air}} = \frac{FAR}{F_s} \quad (24)$$

Note that we have now calculated the engine performance without using the mass flow rate or the nozzle outlet area. We can now adjust the mass flow rate to achieve the required thrust F_t ,

$$\dot{m}_{air} = \frac{F_t}{F_s} \quad (25)$$

Once we have achieved the appropriate mass flow rate, we can calculate the outlet area using the continuity equation. Here we use the total mass flow, including the added fuel:

$$\dot{m}_{total} = \dot{m}_{air}(1 + FAR) \quad (26)$$

Grant

$$A = \frac{\dot{m}_{total}}{(\rho_5 C_5)} \quad (27)$$

This nozzle area will be set on DP and off DP. The nozzle diameter is given by :

$$D = \sqrt{4A/\pi} \quad (28)$$

OFF DESIGN POINT (ODP)

According to C.N.Reffold the use of semi-dimensional mass flow DP (SDMF), $\dot{m}\sqrt{T/P}$, for turbine and nozzle as data to determine CR, TET and turbine temperature ratio (TTR) in off DP calculation. Unfortunately this requires a “nested” iterative process where an estimate is made

on CR followed by iteration of TET and TTR to match compressor power; this is then followed by a fine-grained guess on CR to repeat the cycle over and over. Therefore, the solution process can be time consuming and complicated.

The assumptions used are as follows:

- The compressor and turbine will be choked at all steady-state operating conditions in a single-spool turbojet. Thus, the turbine temperature and pressure will remain constant.
- The adiabatic efficiencies of the compressor and turbine remain constant.
- The ratio of the DP and off-DP semi-dimensional mass flows (SDMF), $\dot{m}\sqrt{T}/P$, is constant and is proportional to the square of the ratio of the DP and off-DP semi-dimensional shaft speeds, N/\sqrt{T} .

Off design point specifications

The off DP temperature, pressure and Mach number are specified. This allows the calculation of P_1 , T_1 and TAS (Ca) using equations (1)-(5). The Off-DP mass flow is found using P_1 and T_1 , by the following method:

$$\dot{m}\sqrt{T}/P = kN^2/T$$

Thus

$$\left(\frac{\dot{m}T^{3/2}}{pN^2}\right)_{off\ DP} = k = \left(\frac{\dot{m}T^{3/2}}{pN^2}\right)_{DP} \quad (29)$$

$$\dot{m}_{off\ DP} = \dot{m}_{DP} \left(\frac{T_{DP}}{T_{off\ DP}}\right)^{3/2} \left(\frac{P_{off\ DP}}{P_{DP}}\right) \left(\frac{N_{off\ DP}}{N_{DP}}\right)^2 \quad (30)$$

Cycle calculation

The SDMF of this DP compressor, $\dot{m}\sqrt{\Delta T}/P_2$, and TTR must be calculated. These must remain the same outside the DP; changes in the CR estimate will be made to balance the SDMF. An estimate of TET will be made to equalize the TTR.

Once the mass flow is known, the temperature and pressure ratios for the cycle can be found; iterative method in excel (Microsoft office with solver). With the CR and TET estimates entered into the program, the temperature and pressure changes for the compressor and turbine can be clearly, P_1 is multiplied by CR to yield P_2 . The compressor temperature rise is calculated using the adiabatic efficiency (equation 8). Thus:

$$T_2 = T_1 + \frac{T_1}{\eta_{ca}} \left[\left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} - 1 \right] \quad (31)$$

TET (T_3) is selected and P_3 is found by equation (9). It is not necessary to calculate FAR until the final TET figure is obtained. The temperature drop across the turbine is determined by referring to the specific power of the compressor:

$$T_4 = T_3 - \frac{C_{pc}(T_2 - T_1)}{C_{pt}\eta_{shaft}} \quad (32)$$

The pressure drop for the turbine is calculated as follows:

$$P_4 = P_3 \left[1 - \frac{1 - T_4/T_3}{\eta_{ta}} \right]^{\gamma/\gamma-1} \quad (33)$$

Given T_2 , P_2 , T_4 and P_4 , the SDMF is evaluated and a new CR is estimated until the off and DP values are nearly equal.

When the CR is solved, the TET is found by iteration until the off and DP TTR is match within acceptable limits.

4. RESULTS AND DISCUSSION

After selecting the appropriate configuration, the engine parameters listed in Table 1. Under various flight conditions, and the specific fuel consumption will vary due to changes in air mass flow due to density differences and variations in momentum resistance with forward speed. In addition, even though the engine is running at a constant rotational speed, the turbine entry temperature will change according to with intake conditions. Typical variations of thrust and specific fuel consumption with altitude for a simple turbojet gas turbine are shown in table 1 Based on increasing altitude due to the beneficial effect of lower inlet temperature. However, the specific fuel consumption shows an increase with increasing altitude, as shown in Table 1. In addition, the specific fuel consumption is dependent on the ambient temperature. From the variation of thrust and specific fuel consumption, it is clear that the fuel consumption will be greatly reduced at high altitudes.

TABLE 1. DATA DETAIL AND RESULTS

Parameter type	Parameter	Turbojet			
		DP	ODP-1	ODP-2	ODP-3
Ambient Parameters	Altitude	100 m	30	50	70
	Ambient Temperature T_a	287.35 K	287.81	287.68	287.55
	Ambient Pressure Pa	Bar	1.01	1.01	1.00
	Ambient Density	kg/m^3	0.7363	0.7363	0.7363
Parameter of the Gas Turbine	Speed	0.8 M	0.3	0.5	0.7
	Compressor Pressure Ratio CR	3.99	4.57	4.45	4.17
	Turbine Entry Temperature T_3	1078 K	1093	1102	1090
	Efficiency for Shaft η_s	0.99			
	Polytropic Efficiency for Compressor η_c	0.91			
	Intake PR max	0.97			
	Efficiency for Combustion η_B	0.98			
	Polytropic Efficiency for Turbine η_T	0.88			
	Combustor OPL/%	4			
	Other Parameters	Specific Heat at Constant Pressure (Air) C_{pa}	1.005 kJ/(kg K)		
Specific Heat at Constant Pressure (Combustion Gas) C_{pg}		1 kJ/(kg K)			
Specific Heat Ratio of Air γ		1.148			
Specific Heat Ratio of Combustion Gases γ_G		1.333			
Gas Constant R		287 J/(kg K)			
Fuel Calorific Value (LHV)		43000 kJ/kg			
Performance results	Comp. Specific power / \dot{w}_{ac}	178.65 kW/kg	181.06	182.64	180.63
	Comp. adiabatic efficiency η_{ca}	0.89	0.88	0.88	0.88
	Fuel/air ratio	0.0159	0.0170	0.0170	0.0164
	Turbine adiabatic efficiency η_{st}	0.89	0.89	0.89	0.89
	Exit density	0.661 kg/m^3	0.5248	0.564	0.62
	Exit velocity	549.570 m/s	553.2668	555.688	552.62
	F_s	414.302 Ns/kg	518.4720	480.768	437.25
	SFC	0.138 kg/hN	0.1182	0.128	0.13
	F_r	1.181 kN	1.1799	1.180	1.181
	Enter mass flow air = \dot{m}_a	2.851 kg/s	2.2757	2.454	2.70
	Fuel flow	0.045 kg/s	0.0388	0.042	0.04
	F_t	1.200 kN	1.2000	1.200	1.20
	Exit area nozzle	0.008 m^2	0.0080	0.008	0.01
	Diameter nozzle	0.101 m	0.1007	0.101	0.10
	\dot{m}_{total}	2.896 kg/s	2.3145	2.496	2.74
	SDMF β	6.392995	6.392995	6.392995	6.3929956
	TTR	0.8541873	0.8541873	0.8541873	0.8541873

5. CONCLUSION

1. The results of this calculation are used as a reference for designing the main components (compressor, combustion chamber, turbine, and nozzle) of the turbojet engine.
2. Manual calculations of DP and ODP for the turbojet engine used in this study provide a 90% confidence level to produce a thrust target of 1.2 kN.
3. The off DP method used in this paper uses a semi-dimensional mass flow parameter, $\dot{m}\sqrt{\Delta T/P_2}$, to determine the compressor pressure ratio. This method avoids the use of nested iterative calculations and simplifies the solution process.

REFERENCES

- Andrea LAZZARETTO and Andrea TOFFOLO (2001). Analytical and Neural Network Models for Gas Turbine Design and Off-Design Simulation. Int.J. Applied Thermodynamics Vol.4, (No.4), pp.173-182
- C.N.Reffold (1995), Spreadsheet Modelling of Turbojet Performance, Int.J.Engng Ed. Vol.11 No.2 pp.103-110
- H.I.H. Saravanamuttoo , G.F.C. Rogers, H. Cohen, P.V Straznicky, A.C Nix (2017). Gas Turbine Theory (7th ed), Pearson Education Limited, United Kingdom
- <https://artikel-teknologi.com/siklus-brayton/>
- https://commons.wikimedia.org/wiki/File:Turbojet_operation_centrifugal_flow-en.svg
- Jack D. Mattingly, William H. Heiser, David T. Pratt (2002). Aircraft Engine Design (2nd ed), American Institute of Aeronautics and Astronautics, Inc
- Klein, D., & Abeykoon, C. (2015). Modelling of a turbojet gas turbine engine. In host publication (pp. 198-204)
- Mustafa Cavcar. The International Standard Atmosphere (ISA), Anadolu University, 26470 Eskisehir, Turkey